Scaling Index Method as novel technique for assessment of local topology

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Aims

Assessment and proper description of the local topological and morphological characteristics of materials with porous and irregular architecture is of a great importance for many scientific and engineering studies. Deterioration of the structure with time, deformations and loss of strength under the load and other processes lead to the topological changes inside the materials and need both qualitative analysis and quantitative evaluation. In present work we introduce Scaling Index Method (SIM), a novel tool for description of the local topology of an arbitrary structure by indication of the one-dimensional (rode-like) and two-dimensional (plate-like) elements of structure. We apply numerical technique based on the SIM to 3D μCT grey value images of the trabecular bone specimens taken from human vertebrae in vitro. Combination of the SIM with finite element analysis (FEA) on tissue level gives a possibility to combine redistribution of the stresses with topological characteristics of the structure elements.

Method

The Scaling Index Method[1-3] (SIM) is well suited for quantifying geometrical aspects on a local level, especially to discriminate rod-like, sheet-like and unstructured (background) image components (fig. 1).

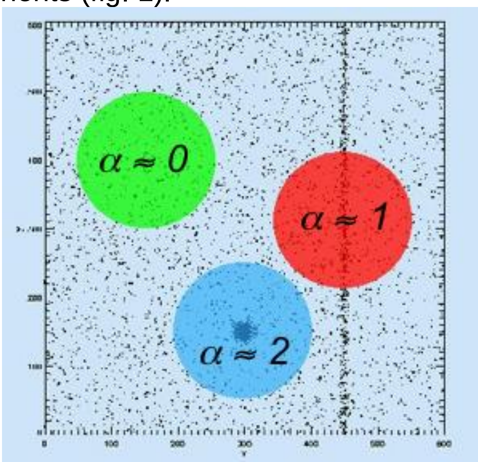

Figure 1. Discrimination between background (green circle with $\alpha\approx 0$), one-dimensional rode-like structure (red circle with $\alpha\approx 1$) and two-dimensional plate-like structure (blue circle with $\alpha\approx 2$).

In SIM a 3D μCT image is described as a set of points with spatial coordinates *x, y, z*. For each voxel the logarithmic gradients

$$\alpha_i = \frac{\partial \log \rho(\vec{p}_i, r)}{\partial \log r}$$

which are called scaling indices, are calculated by means of a Gaussian shaped weighted cumulative point distribution

$$\rho(\vec{p}_i, r) = \sum_{j=1}^{N_{voxel}} e^{-\left(d_{ij}/r\right)^2}$$

where d_{ij} indicates a distance measure between two points in the 3D space. It is the exponential shape of the weighted function ρ that causes SIM to be a local method: the value of the scaling index depends only on the number of neighbours in a small vicinity of the point for which α is calculated. For the case of isotropic scaling indices we use the Euclidean distance between two points in the 3D space

$$d_{ij} = \left\| \vec{p}_i - \vec{p}_j \right\|_2 .$$

Anisotropic scaling indices are calculated by using a generalized quadratic distance measure in form

$$d_{ij}^2 = \lambda_x \left(x_i - x_j\right)^2 + \lambda_y \left(y_i - y_j\right)^2 + \lambda_z (z_i - z_j)^2,$$

where λ_x, λ_y, λ_z are the weighting factors of the tree orthogonal spatial directions, respectively. When all directions have the same weight, i.e. $\lambda_x = \lambda_y = \lambda_z$ the distance measure becomes the isotropic Euclidean one. In the case of human vertebrae with natural vertical loading along z-axis we set $\lambda_x = \lambda_y = 5\ \lambda_z = 1$.

Results

In present study we apply SIM to μCT images of trabecular network taken from the human vertebrae. The scans were acquired for the central 6 mm in length of the specimen using a μCT scanner (Scanco Medical, Bassersdorf, Switzerland). The resulting μCT grey-value images with isotropic spatial resolution of 26 mm were segmented using a fixed global threshold equal to 22% of the maximal grey value to extract the mineralised bone phase. These binarized 3D images (fig. 2) are used for subsequent numerical analysis.

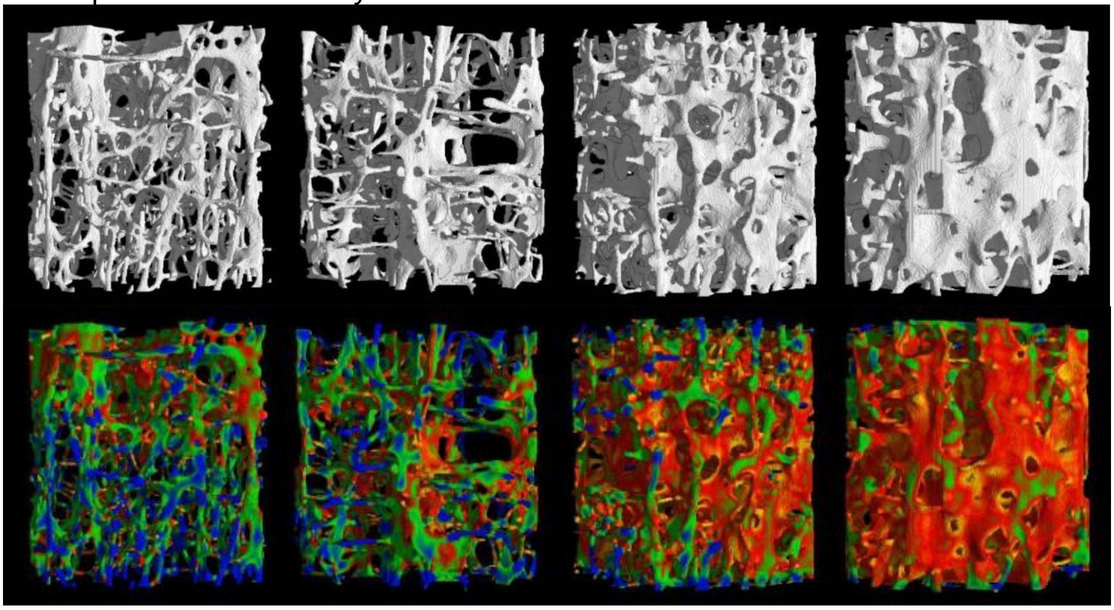

Figure 2. 3D μCT images (upper row) and α-representation (lower row) of bone specimens with different bone fraction (*BV/TV*) and maximum compressive strength *MCS*. From left to right: *BV/TV=0.07* and *MCS=17.87*; *BV/TV=0.07* and *MCS=82.53*; *BV/TV=0.17* and *MCS=90.84*; *BV/TV=0.17* and *MCS=157.00*.

The values of scaling indices can be compiled in the probability density function (pdf) $P(\alpha) = prob\left(\alpha \in [\alpha, \alpha + \Delta\alpha]\right)$. Analysing the $P(\alpha)$ spectrum of the structure (fig. 3, upper row) one can distinguish between strong and weak bones. For specimens with strong trabecular structure the position of the maximum of the $P(\alpha)$ distribution is typically shifted to higher values of α. This shift reflects the fact that strong bones have more plate-like structures than weak ones. In order to understand the redistribution of the deformation energy accumulated inside the trabecular structure during compressive loading we can combine SIM with FEA. Both FEA and SIM propose alternative representation of the bone structure on tissue level. Each voxel can be characterised by two new properties: effective strain ε_{eff} obtained by FEM and scaling index α obtained by SIM. We calculate the average effective strain ε_{eff} for voxels having the same topological values of scaling indices (fig. 3, lower row). Both in strong and weak bones maximum average effective strain is accumulated in substructures with $\alpha \approx 2.5$, which correspond to plate-like trabeculae, but according to the $P(\alpha)$ spectrum the amount of plates in weak bones is smaller than in strong ones. This means that plates are the main load bearing substructure of the trabecular network and the relative amount of plates to roads plays the main role for bone strength and stability on a global level.

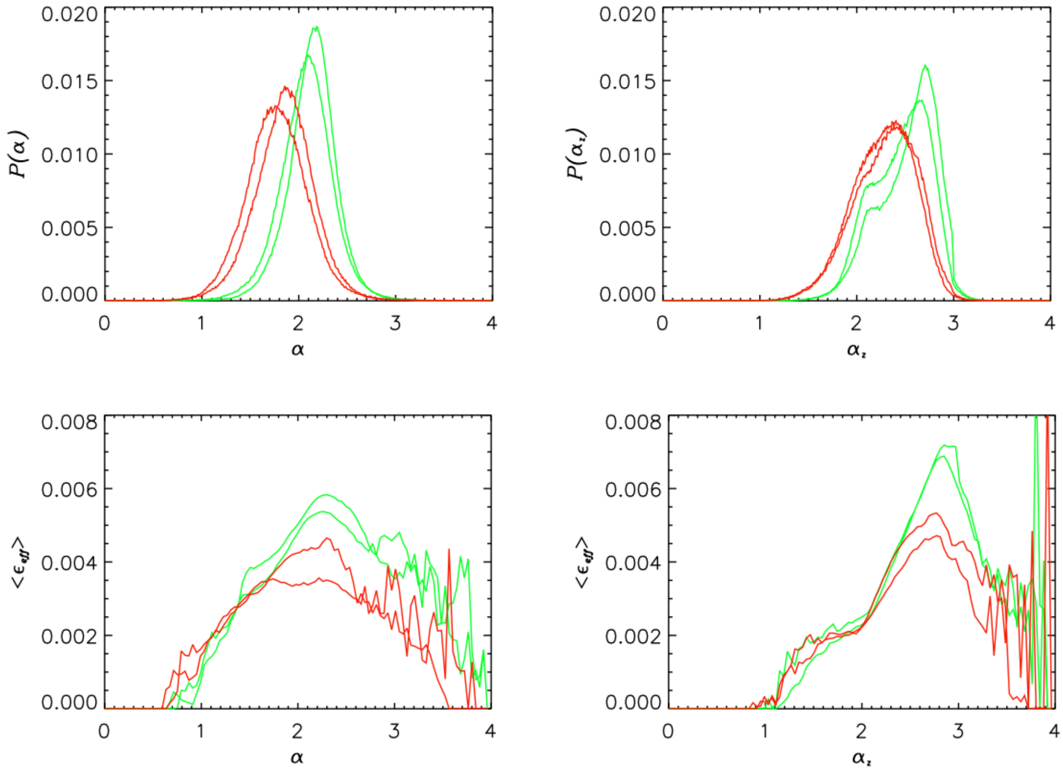

Figure 3. Probability distribution function (upper row) and effective strain ε_{eff} (calculated with FEM) averaged over the voxels with the same value of scaling indices (left: isotropic, right: anisotropic). Red lines: weak bones with *BV/TV=0.07*; green lines: strong bones with *BV/TV=0.17*.

Conclusion

Scaling Index Method proposes a new qualitative analysis and quantitative evaluation of the local topology of the materials with porous and irregular structure. Combination of the SIM with FEA leads to the better understanding of stress redistribution between elements with different topological characteristic. By averaging values of ε_{eff} over voxels with the same topological characteristics α one can derive the dependence of biomechanical stresses in the bone from the topology of the underlying structure.

References:

1. Räth C and Morfill G 1997 *Journ. Opt. Soc. Am.* A **14** 3208-15
2. Monetti R, Boehm H, Mueller D, Rummeny E, Link T and Räth C 2004 *Proc. SPIE* **5370** 215-24
3. Räth C, Monetti R, Bauer J, Sidorenko I, Mueller D, Matsuura M, Lochmüller E-M, Zysset P and Eckstein F 2008 *New J. Phys.* **10** 125010-18